L/E-Flatness of the Electron-Like Event Ratio in Super-Kamiokande and a Degeneracy in Neutrino Masses

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Abstract

We show that the L/E-flatness of the electron-like event ratio in the Super-Kamiokande atmospheric neutrino data implies the equality of the expectation values for the muon and tau neutrino masses. We establish this result by obtaining a set of three constraints on the neutrino-oscillation mixing matrix as contained in the indicated flatness. The resulting 3×3 neutrino-oscillation matrix depends only on one angle. A remarkable result that follows directly from this matrix is the consistency between the mixing angles observed by LSND and Super-Kamiokande.

1 Introduction

One of the most remarkable features of the latest Super-Kamiokande data on the atmospheric neutrino anomaly is the L/E-flatness of the electron-like event ratio [1]. Given the definitions,

$$\mathcal{R}_e \equiv \frac{\text{Experimentally observed } e\text{-like events}}{\text{Theoretically expected } e\text{-like events (without } \nu \text{ oscillations)}}, \quad (1)$$

$$\mathcal{R}_\mu \equiv \frac{\text{Experimentally observed } \mu\text{-like events}}{\text{Theoretically expected } \mu\text{-like events (without } \nu \text{ oscillations)}}, \quad (2)$$

the Super-Kamiokande data reveals a significant L/E dependence for $\mathcal{R}_\mu$, while $\mathcal{R}_e$ is consistent with unity with no L/E dependence. In this Letter, following an earlier study [2], we establish that this L/E flatness of $\mathcal{R}_e$, consistent with $\mathcal{R}_e = 1$, constrains the neutrino mixing matrix dramatically. All resulting neutrino-oscillation matrices are equivalent to each other, under appropriate redefinitions of the underlying mass eigenstates. They all carry the property that the expectation values of the muon and tau neutrino masses be equal. Moreover, the mixing matrices derived in this Letter naturally lead to a consistency between the mixing angles observed by LSND and Super-Kamiokande. This circumstance makes us suspect that the solar neutrino anomaly points towards a richer phenomenology, perhaps beyond neutrino masses, in the physics of neutrino oscillations.

2 Constraints on the Neutrino-Oscillation Matrices

Let us assume that at the top of the atmosphere, at $t = 0$, the number of ν_e and ν_μ produced is N_e and N_μ, respectively. Although both neutrinos and antineutrinos are produced in both flavours, we shall use the terms "electron neutrinos" and "muon neutrinos" loosely, to include both the ν and $\bar{\nu}$. In general, the ratio of ν_μ to ν_e neutrinos,

$$r = \frac{N_\mu}{N_e} \quad (3)$$

is a function of energy. However, for the relevant energy range in Super-Kamiokande, it may be assumed constant (as shall be done in this Letter). Within a detector at a distance $L \simeq t$ from the production point, the number of electron neutrinos, N'_e, is given by

$$N'_e = N_e P_{ee} + N_\mu P_{\mu e}, \quad (4)$$

where P_{ee} and $P_{\mu e}$ are the neutrino oscillation probabilities $P(\nu_e \rightarrow \nu_e)$ and $P(\nu_\mu \rightarrow \nu_e)$, respectively. Assuming that the underlying mass eigenstates are relativistic [3], the neutrino oscillation probability $P(\nu_\ell \rightarrow \nu_{\ell'})$ takes the form

$$
\begin{aligned}
P(\nu_\ell \rightarrow \nu_{\ell'}) = {}& \delta_{\ell\ell'} \\
& - 4\, Re(U_{\ell'1}U_{\ell 1}^* U_{\ell'2}^* U_{\ell 2}) \sin^2(\varphi_{12}) + 2\, Im(U_{\ell'1}U_{\ell 1}^* U_{\ell'2}^* U_{\ell 2}) \sin(2\varphi_{12}) \\
& - 4\, Re(U_{\ell'1}U_{\ell 1}^* U_{\ell'3}^* U_{\ell 3}) \sin^2(\varphi_{13}) + 2\, Im(U_{\ell'1}U_{\ell 1}^* U_{\ell'3}^* U_{\ell 3}) \sin(2\varphi_{13}) \\
& - 4\, Re(U_{\ell'2}U_{\ell 2}^* U_{\ell'3}^* U_{\ell 3}) \sin^2(\varphi_{23}) + 2\, Im(U_{\ell'2}U_{\ell 2}^* U_{\ell'3}^* U_{\ell 3}) \sin(2\varphi_{23}).
\end{aligned}
\tag{5}
$$

The kinematic phases φ_{ij}, which appear in the above expression, are defined as

$$
\varphi_{ij} = 1.27 \Delta m_{ij}^2 \frac{L}{E}.
\tag{6}
$$

In the above equation, E is the neutrino energy (expressed in MeV), L is the distance between the generation point and the detection point (expressed in meters), and $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ (expressed in eV^2). For Dirac neutrinos, the matrix U is a 3×3 unitary matrix, parameterized in terms of three mixing angles (θ, β, ψ) and a CP-violating phase, δ:

$$
U = \begin{pmatrix}
c_\theta c_\beta & s_\theta c_\beta & s_\beta \\
- c_\theta s_\beta s_\psi e^{i\delta} - s_\theta c_\psi & c_\theta c_\psi - e^{i\delta} s_\theta s_\beta s_\psi & c_\beta s_\psi e^{i\delta} \\
- c_\theta s_\beta c_\psi + s_\theta s_\psi e^{-i\delta} & - s_\theta s_\beta c_\psi - c_\theta s_\psi e^{-i\delta} & c_\beta c_\psi
\end{pmatrix}
\tag{7}
$$

in the Maiani representation [4]. We use the abbreviated notations $c_\theta = \cos\theta$, $s_\theta = \sin\theta$, etc., and we shall henceforth set the CP-violating phase $\delta = 0$. Assuming that the Super-Kamiokande data for electron-like events,

$$
\mathcal{R}_e = \frac{N_e'}{N_e},
\tag{8}
$$

is unity over its relevant L/E range (an assumption which is certainly valid within the systematic and statistical errors) implies that

$$
P_{ee} + r P_{\mu e} = 1.
\tag{9}
$$

Furthermore, from the unitarity condition, we have

$$
P_{ee} + P_{e\mu} + P_{e\tau} = 1.
\tag{10}
$$

Consequently, for a vanishing CP-violating phase $\delta = 0$ one has $P_{e\mu} = P_{\mu e}$, and we obtain

$$(r - 1)P_{e\mu} = P_{e\tau}. \tag{11}$$

Using the explicit expressions for the oscillation probabilities $P_{e\mu}$ and $P_{e\tau}$ yields

$$
\begin{aligned}
&U_{e1}U_{e2}\left[(r-1)U_{\mu 1}U_{\mu 2} - U_{\tau 1}U_{\tau 2}\right]\sin^2(\varphi_{12}) + \\
&U_{e1}U_{e3}\left[(r-1)U_{\mu 1}U_{\mu 3} - U_{\tau 1}U_{\tau 3}\right]\sin^2(\varphi_{13}) + \\
&U_{e2}U_{e3}\left[(r-1)U_{\mu 2}U_{\mu 3} - U_{\tau 2}U_{\tau 3}\right]\sin^2(\varphi_{23}) = 0.
\end{aligned}
\tag{12}
$$

Since this condition should hold for all relevant values of L/E, we obtain the following system of three equations with three unknowns (θ, β, and ψ):

$$U_{e1}U_{e2}\left[(r-1)U_{\mu 1}U_{\mu 2} - U_{\tau 1}U_{\tau 2}\right] = 0, \tag{13}$$
$$U_{e1}U_{e3}\left[(r-1)U_{\mu 1}U_{\mu 3} - U_{\tau 1}U_{\tau 3}\right] = 0, \tag{14}$$
$$U_{e2}U_{e3}\left[(r-1)U_{\mu 2}U_{\mu 3} - U_{\tau 2}U_{\tau 3}\right] = 0. \tag{15}$$

In the following we shall first investigate the possible solutions of this system of equations. We then obtain the advertised result on the neutrino-mass degeneracy. Finally, we briefly study the compatibility of the resulting neutrino-oscillation mixing matrix for the LSND and Super-Kamiokande experiments, as well as its consequences for the solar neutrino deficit.

3 The Resulting Neutrino-Oscillation Mixing Matrices

One possible class of solutions to the system of equations above is given by requiring that the expressions in the square brackets in Eqs.(13)–(15) be zero simultaneously. However, this leads to a rather uninteresting class of mixing matrices, namely the unit matrix and others that are basically equivalent to it, under appropriate redefinitions of the mass eigenstates. Hence, the problem of the L/E flatness in $\mathcal{R}_e$ is trivially solved, but there would be no neutrino oscillations either. This would contradict the existing data and we therefore discard such solutions.

A more interesting class of solutions follows when one of U_{e1}, U_{e2}, or U_{e3} is zero. This determines one of the mixing angles and two of the three equations are trivially satisfied. The remaining equation fully determines a second mixing angle. We discuss this class of solutions below.

3.1 The $U_{e1} = 0$ Case

Since $U_{e1} = c_\theta c_\beta$, one solution to $U_{e1} = 0$ is $c_\beta = 0$. However, this implies that both U_{e1} and U_{e2} vanish identically, which in turn means that there are no $\nu_e \to \nu_\mu$ and no $\nu_e \to \nu_\tau$ oscillations. We shall, therefore, discard this solution, as it is not of interest in the context of the existing data. The other solution to $U_{e1} = 0$ is $c_\theta = 0$, which implies $s_\theta = \pm 1$. Considering only the $s_\theta > 0$ solution (as an illustration), the mixing matrix becomes:

$$
U = \begin{pmatrix} 0 & c_\beta & s_\beta \\ -c_\psi & -s_\beta s_\psi & c_\beta c_\psi \\ s_\psi & -s_\beta c_\psi & c_\beta c_\psi \end{pmatrix}.
\tag{16}
$$

From Eq.(15) we have

$$
s_\beta c_\beta [(r-1)s_\psi^2 - c_\psi^2] = 0,
\tag{17}
$$

with two trivial solutions, $s_\beta = 0$ and $c_\beta = 0$. They imply that $U_{e1} = 0$ and $U_{e3} = 0$, or $U_{e1} = 0$ and $U_{e2} = 0$, respectively. Both cases lead to no $\nu_e \to \nu_\mu$ and no $\nu_e \to \nu_\tau$ oscillations, and are discarded as discussed above. More generally, however:

$$
s_\psi = 1/\sqrt{r},
\tag{18}
$$
$$
c_\psi = \sqrt{r-1}/\sqrt{r}.
\tag{19}
$$

Notice that solutions are allowed for both $s_\psi < 0$ and $c_\psi < 0$, but we shall restrict our (illustrative[1]) discussion solely to the $s_\psi > 0$ and $c_\psi > 0$ case. At this point we explicitly set $r = 2$ and thus the full mixing matrix becomes:

$$
U = \begin{pmatrix} 0 & c_\beta & s_\beta \\ -1/\sqrt{2} & -s_\beta/\sqrt{2} & c_\beta/\sqrt{2} \\ 1/\sqrt{2} & -s_\beta/\sqrt{2} & c_\beta/\sqrt{2} \end{pmatrix}.
\tag{20}
$$

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[1] The remaining U's can be easily enumerated by the reader.

3.2 The $U_{e2} = 0$ Case

Since $U_{e2} = s_\theta c_\beta$, one solution to $U_{e2} = 0$ is $c_\beta = 0$, which is discarded as discussed above. The $s_\theta = 0$ solution implies $c_\theta = \pm 1$, and considering only the $c_\theta > 0$ solution, the mixing matrix becomes:

$$U = \begin{pmatrix} c_\beta & 0 & s_\beta \\ -s_\beta s_\psi & c_\psi & c_\beta s_\psi \\ -s_\beta c_\psi & -s_\psi & c_\beta c_\psi \end{pmatrix}. \tag{21}$$

From Eq.(14) we have:

$$s_\beta c_\beta[(r-1)s_\psi^2 - c_\psi^2] = 0, \tag{22}$$

and disregarding the trivial solutions $s_\beta = 0$ and $c_\beta = 0$, the general mixing matrix yields (for $r = 2$):

$$U = \begin{pmatrix} c_\beta & 0 & s_\beta \\ -s_\beta/\sqrt{2} & 1/\sqrt{2} & c_\beta/\sqrt{2} \\ -s_\beta/\sqrt{2} & -1/\sqrt{2} & c_\beta/\sqrt{2} \end{pmatrix}. \tag{23}$$

3.3 The $U_{e3} = 0$ Case

Since $U_{e3} = s_\beta$, $U_{e3} = 0$ implies simply that $s_\beta = 0$ and thus $c_\beta = \pm 1$. Considering only the $c_\beta > 0$ solution, the mixing matrix becomes:

$$U = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta c_\psi & c_\theta c_\psi & s_\psi \\ s_\theta s_\psi & -c_\theta s_\psi & c_\psi \end{pmatrix}. \tag{24}$$

From Eq.(13) we have:

$$s_\theta c_\theta[s_\psi^2 - (r-1)c_\psi^2] = 0, \tag{25}$$

and disregarding the trivial solutions $s_\theta = 0$ and $c_\theta = 0$, the general solution leads to the following mixing matrix (for $r = 2$):

$$U = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta/\sqrt{2} & c_\theta/\sqrt{2} & 1/\sqrt{2} \\ s_\theta/\sqrt{2} & -c_\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{26}$$

4   The Degenerate Muon and Tau Neutrino Masses

Referring to the obtained neutrino-oscillation matrices U above, and taking note of the definition for the expectation value of the neutrino masses (recall that for $\delta = 0$ all the $U_{\ell j}$ elements are real)

$$\langle m(\nu_\ell) \rangle \equiv \sum_j U_{\ell j}^2 \, m_j, \tag{27}$$

we immediately come to the general conclusion on the mass degeneracy of the muon and tau neutrinos:

$$\langle m(\nu_\mu) \rangle = \langle m(\nu_\tau) \rangle. \tag{28}$$

For the three cases enumerated above, one readily sees that the "degenerate mass" carries the values

$$\frac{1}{2} \left(m_1 + s_\beta^2 m_2 + c_\beta^2 m_3 \right), \tag{29}$$

$$\frac{1}{2} \left(s_\beta^2 m_1 + m_2 + c_\beta^2 m_3 \right), \tag{30}$$

$$\frac{1}{2} \left(s_\theta^2 m_1 + c_\theta^2 m_2 + m_3 \right), \tag{31}$$

respectively. The enumerated mixing matrices U are immediately noted to be equivalent to each other under redefinitions of the underlying mass eigenstates m_1, m_2, and m_3.

5 Implications for the Muon-Like Event Ratio: LSND versus Super-Kamiokande

Apart from the mass degeneracy in the muon and tau neutrino masses, a remarkable result that follows directly from the derived neutrino-oscillation matrices is the consistency between the mixing angles observed by LSND and Super-Kamiokande. We briefly discuss this in the following.

The number of muon neutrinos at the Super-Kamiokande detector, N'_μ, is given by

$$N'_\mu = N_\mu P_{\mu\mu} + N_e P_{e\mu}, \tag{32}$$

and thus, the muon-like event ratio reads

$$\mathcal{R}_\mu = \frac{N'_\mu}{N_\mu} = P_{\mu\mu} + \frac{1}{r} P_{e\mu}. \tag{33}$$

Without loss of generality, let us consider the mixing matrix given by Eq.(26). The ν_μ survival probability, $P_{\mu\mu}$, and the $P_{e\mu}$ oscillation probability read

$$P_{\mu\mu} = 1 - s_\theta^2 c_\theta^2 \sin^2(\varphi_{12}) - s_\theta^2 \sin^2(\varphi_{13}) - c_\theta^2 \sin^2(\varphi_{23}), \tag{34}$$

and

$$P_{e\mu} = 2 s_\theta^2 c_\theta^2 \sin^2(\varphi_{12}), \tag{35}$$

respectively. Therefore, the Super-Kamiokande muon-like event ratio, $\mathcal{R}_\mu$, yields

$$\mathcal{R}_\mu = 1 - s_\theta^2 \sin^2(\varphi_{13}) - c_\theta^2 \sin^2(\varphi_{23}). \tag{36}$$

Here we have explicitly set $r = 2$. At this point one cannot proceed any further without additional information on either the mixing angle θ, or the mass differences Δm_{13}^2 and Δm_{23}^2.

This is the point where the LSND evidence comes into play. As reported in Refs. [5,6], the LSND experiment has obtained evidence for neutrino oscillations in both the $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ channels. Although interpreted in terms of the simpler, two-generations neutrino mixing, the allowed regions obtained by LSND help us gain further insight. Within this framework, the $\nu_\mu \to \nu_e$ oscillation probability is given by

$$P_{\mu e}^{LSND} = \sin^2\left(2\Theta_{LSND}\right)\sin^2\left(\varphi_{12}\right), \tag{37}$$

which is very similar to the expression in Eq.(35). Indeed for the $\bar{\nu}_\mu \to \bar{\nu}_e$ and $\nu_\mu \to \nu_e$ oscillation channels one may effectively identify $\sin^2\left(2\Theta_{LSND}\right)$ with $2s_\theta^2 c_\theta^2 = \frac{1}{2}\sin^2(2\theta)$. Therefore, a very small mixing angle Θ_{LSND}, approximately of $\mathcal{O}(10^{-1})$ – as indeed favored by the allowed regions indicated by LSND – implies a very small mixing angle θ, also of $\mathcal{O}(10^{-1})$, in our formalism. This in turn implies that the muon-like event ratio, $\mathcal{R}_\mu$, reads

$$\mathcal{R}_\mu = 1 - c_\theta^2 \sin^2(\varphi_{23}) + \mathcal{O}(10^{-2}), \tag{38}$$

as opposed to simply

$$\mathcal{R}_\mu = 1 - \sin^2\left(2\Theta_{SK}\right)\sin^2\left(\varphi_{23}\right), \tag{39}$$

if $\mathcal{R}_\mu$ were to be expressed in the two-generations neutrino oscillations formalism, where only $\nu_\mu \to \nu_\tau$ transitions are allowed – as interpreted by the Super-Kamiokande group. Therefore, since $c_\theta \approx 1$, implies that $\sin^2(2\Theta_{SK}) \approx 1$ as well, as indeed reported in Ref. [1].

6   The Solar Neutrino Deficit

If the Super-Kamiokande/LSND consistency is firmly established by future experiments, then the physics of neutrino oscillations shall be found not only to contain massive neutrinos, but may also point towards new physics. This arises from the long-standing solar neutrino deficit, as measured by a variety of experiments, with different sensitivities and detection techniques [7–11]. Within the framework of neutrino oscillations, the ratio of measured to predicted solar neutrinos, R_e, is simply given by the ν_e survival probability, P_{ee}. Using the mixing matrix in Eq.(26), this reads

$$P_{ee} = 1 - 4\, s_\theta^2 c_\theta^2 \, \sin^2(\varphi_{12}), \tag{40}$$

with an underlying mass scale $\Delta m_{12}^2 = \mathcal{O}(1)$ eV^2, as indicated by the LSND experiment. Consequently, the kinematic term $\sin^2(\varphi_{12})$ effectively averages out to 1/2. Furthermore, since the mixing angle θ is of $\mathcal{O}(10^{-1})$, as we have argued in the previous Section, the predicted solar neutrino ratio is practically $R_e = 1$, i.e., no solar neutrino deficit. This is obviously in disagreement with the measured solar neutrino ratio of $R_e \approx 0.5$, as reported by the above-mentioned experiments. A popular solution to the solar neutrino deficit conjectures the existence of a sterile neutrino(s). This may very well be the way

nature is. However, before the sterile neutrino solution is invoked, one must make a fundamental observation that the flavour and mass measurements do not commute [2]. This incompatibility of the flavour and mass measurements can lead to a violation of the principle of equivalence, which in turn modifies the standard neutrino oscillation phenomenology in a fundamental manner [12]. Such a violation of the principle of equivalence will take us into new physics. At the same time, this might provide an elegant solution to the solar neutrino anomaly [2,12–16].

7 Conclusions

We conclude that L/E flatness of the electron-like event ratio in the Super-Kamiokande data on atmospheric neutrinos implies a mass degeneracy for the muon and tau neutrino. The obtained results support recent considerations on maximal mixing, bi-maximal mixing, and the degenerate neutrino masses [18–21]. More precise data on the discussed L/E flatness would be most helpful to settle the insights gained in this Letter. If the L/E flatness of $\mathcal{R}_e$ is firmly established in the future by the data, then one would be able to severely constrain the theoretical models for neutrino masses and neutrino oscillations. The general cases considered by us require that one of the U_{ej} vanishes. This result is in agreement with the conclusions reached by several authors, see e.g. Ref. [22]. Since all neutrino-oscillation matrices obtained by us are physically equivalent, we have arrived at a unique 3×3 neutrino-oscillation mixing matrix that depends only on one angle. This matrix clearly shows the consistency between the mixing angles observed by the LSND and Super-Kamiokande experiments.

We explicitly note that the constraint implied by the L/E flatness of the Super-Kamiokande e-like event ratio, as contained in this Letter, is a generalized version of that contained in Ref. [2]. However, this is not the main purpose of this Letter. Within the standard three-neutrino oscillation framework, the mixing matrices that we now obtain exhaust all possibilities consistent with the Super-Kamiokande implied constraint. In particular, we show that the Super-Kamiokande inferred $\nu_\mu \leftrightarrow \nu_\tau$ oscillation (with a complete decoupling from the ν_e oscillations) is too strong a conclusion. The class of solutions consistent with the Super-Kamiokande data, as systematically derived and analyzed here, is significantly richer and in particular leaves important room for oscillations away from, and into, the ν_e channel. In Ref. [2] only a very specific solution was obtained. This Letter obtains a class of new non-trivial and physically interesting solutions. In particular, the LSND and Super-Kamiokande compatibility (as contained in bins where L/E for LSND is close to that for Super-Kamiokande) emerges as a significant new result. With this compatibility established, it should now be clear to the neutrino-oscillation

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community that Super-Kamiokande and LSND have an overlapping and mutually consistent regime in the neutrino-oscillation parameter space. Thus, either something in the non-overlapping regime of the Super-Kamiokande and LSND results must change to accommodate the solar neutrino anomaly – or, we must accept seriously that some new physics is hinted at.

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